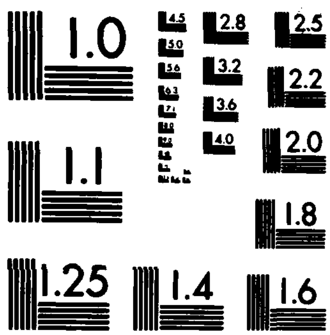
AD-A127 896 DOUBLE AUCTIONS(U) STANFORD UNIV CA INST FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES R WILSON DEC 82 TR-391 N00014-79-C-0685 1/1

UNCLASSIFIED F/G 5/3 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

DOUBLE AUCTIONS

by

Robert Wilson

Technical Report No. 391
December 1982

PREPARED UNDER

NATIONAL SCIENCE FOUNDATION GRANT SES-81-08226

and the

CENTER FOR RESEARCH ON ORGANIZATIONAL EFFICIENCY
STANFORD UNIVERSITY

Contract ONR-N00014-79-C-0685, United States Office of Naval Research

THE ECONOMICS SERIES

INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
Fourth Floor, Encina Hall
Stanford University
Stanford, California
94305

DOUBLE AUCTIONS*

by

Robert Wilson

1. Introduction

In practice, markets are organized in many different ways. They differ in the trading rules that regulate the exchange process, and in the special roles played by intermediaries such as brokers and specialists. In every case, however, the terms of trade ultimately depend upon the actions chosen by the participants. Each agent therefore has an incentive to take account of this dependence in deciding on the trading strategy to use.

Much of modern economic theory in the Walrasian mold, nevertheless, has been based on a model that assumes that buyers and sellers respond to prices that are known beforehand to clear the market. Though implausible, this model is at least consistent: if the agents' preferences and endowments are common knowledge then (under suitable regularity assumptions) there exist prices that, if calculated and fixed beforehand, would achieve equality between the buyers' demands and the sellers' supplies. Or, if the agents have enough experience to anticipate fairly accurately the clearing price then this model may be a good predictor, as indeed it has been in many of the experiments that have been conducted (Smith [1981]).

*This research was supported by the Office of Naval Research Grant ONR-N00014-79-C-0685 at the Center for Research on Organizational Efficiency, Institute for Mathematical Studies in the Social Sciences at Stanford University, and National Science Foundation Grant SES-81-08226 to the Graduate School of Business, Stanford University.

A fundamental theory of markets requires a better approximation than the Walrasian model provides. There are at least two desiderata. One is that an agent's preferences and perhaps endowment are not likely to be known to others. A second is that the numbers of buyers and sellers need not be so large as to yield "perfect competition." That is, the theory provides a better approximation to a wider range of phenomena if it encompasses trading among a few participants with incomplete information. This agenda is followed in the recent work that models the trading process as a noncooperative game with incomplete information. In this approach the criterion of Nash equilibrium among the agents' trading strategies is used to predict the outcome. That is, each agent's strategy is supposed to be an optimal response to the others' strategies.

Studying every possible set of trading rules in this way poses an enormous task, however. It is best to study first those trading rules that are important in practice or that have salient theoretical properties.

In this paper I study an example of a sealed-bid double auction. An auction of this kind institutionalizes the Marshallian conception of intersecting demand and supply curves. The rules specify that each buyer or seller submits a sealed bid or offer. These bids and offers are then arrayed into aggregate demand and supply curves and a clearing price is selected from their intersection. Those bids (offers) above (below) the clearing price are executed at the clearing price.

I choose sealed-bid double auctions to study in part because they are amenable to quite simple analysis. As we shall see, though, many of the results are applicable to a wider class of trading rules: the theory of incentive compatibility is used to establish that the main properties depend on the agents' chances of executing a trade. For the example studied, a sealed-bid double auction is an efficient trading process. Other trading processes that give the agents the same chances of executing a trade are also efficient. The results derived here might, therefore, have applications to trading rules found more commonly in practice. The prime candidate is the oral double auction used in experimental studies (Smith, Williams, Bratton, amd Vannoni [1982], Plott and Smith [1978], Easley and Ledyard [1981]).

Our analysis is derived from the insightful studies of the one-buyer, one-seller case by Myerson and Satterthwaite [1981] and by Chatterjee and Samuelson [1979]. Section 2 formulates the model; Section 3 reviews the main results established in earlier studies, appropriately generalized to the case of multiple buyers and sellers, and develops the sufficient condition for a double auction to be an efficient trading process. Section 4 analyzes an example for which it is established that a double auction is indeed an efficient trading process. The case of many buyers and sellers is examined in Section 5. Some concluding remarks are included in Section 6.

2. Formulation

We consider a market in which two homogenous goods are traded. One is finely divisible and interpreted as money; the other occurs in

indivisible units, or items, any one of which is a perfect substitute for any other. There are $m + n$ agents, of whom m are buyers indexed by $i \in M$ and n are sellers indexed by $j \in N$. Each buyer i has an inelastic demand for one item at a valuation $u_i \in U$, and if he receives x_i items and pays p_i then his net payoff is $x_i u_i - p_i$. Similarly, each seller j has an inelastic supply of one item at a valuation $v_j \in V$, and if he gives up y_j items and receives q_j then his net payoff is $q_j - y_j v_j$. The agents' preferences are devoid of risk aversion and income effects, and each buyer has an ample endowment of money. A feasible allocation requires that each $x_i, y_j \in \{0,1\}$ and

$$\sum_i x_i = \sum_j y_j \quad , \quad \sum_i p_i = \sum_j q_j \quad .$$

Each agent knows whether he is a buyer or a seller and he knows his valuation; any other agent knows only the probability distribution of this valuation. The agents' valuations are distributed independently. The buyers' valuations are distributed identically on an interval $U = [u^o, u^1]$ with a distribution function F having a positive density f. Similarly, the sellers' valuations are distributed identically on an interval $V = [v^o, v^1]$ with a distribution function G having a positive density g.

All of the foregoing, as well as the trading rules to be specified subsequently, are common knowledge among the agents. That is, they define the rules of the game.

In general, a trading rule specifies a set R_i or S_j of feasible actions for each buyer i or seller j, and a function

$$\tau: \quad (X_{i\in M}R_i)X(X_{j\in N}S_j) \rightarrow A \subseteq M(\{\{0,1\}X\underset{\sim}{R}\}^{M\cup N}),$$

that maps combinations of actions into the collection A of probability distributions over the set of feasible allocations. With such a trading rule a strategy for a buyer i or a seller j is a function

$$\rho_i: \ U \rightarrow R_i, \quad \text{or} \quad \sigma_j: \ V \rightarrow S_j \quad ,$$

specifying his action contingent on his valuation. Here we are mainly concerned with <u>revelation games</u> in which $R_i = U$ and $S_j = V$. A <u>direct</u> revelation game has the further property that one obtains a Nash equilibrium in which the strategies are identity functions. It is easily verified that if (ρ,σ) is a strategy combination that is a Nash equilibrium for the game with trading rule τ then the composition $\hat{\tau} = \tau \circ (\rho,\sigma)$ yields a direct revelation game.

A (sealed-bid) double auction is a revelation game with the following trading rule. Let $r[k]$ be the k-th largest of the buyers' bids and $s[k]$ the k-th smallest of the sellers' offers, where $r[k] = -\infty$ if $k > m$ and $s[k] = \infty$ if $k > n$. Specify

$$p = [r[K] + s[K]]/2, \text{ where } K = \max\{k \mid r[k] > s[k]\} \quad .$$

Then

$$(x_i,p_i) = (1,p) \quad \text{if} \quad r_i > p \quad \text{and} \quad (0,0) \quad \text{otherwise} \quad ,$$

and

$$(y_j,q_j) = (1,p) \quad \text{if} \quad s_j < p \quad \text{and} \quad (0,0) \quad \text{otherwise.}$$

That is, all trades are executed at the price that is the midpoint of the interval of prices that will clear the market.

3. Direct Revelation Games

In this section we review some of the properties of direct revelation games that can be found in Myerson and Satterthwaite [1981] for example.

To ensure that the problem is nontrivial assume that $u^1 > v^0$ so that *ex ante* there is some prospect of gains from trade.

Let $U_i(u)$ be the expected payoff of buyer i conditional on his valuation being $u_i = u$. This can be written as $U_i(u) = uP_i(u) - R_i(u)$ where, with the same conditioning, $P_i(u)$ is his probability of acquiring an item and $R_i(u)$ is his expected payment. Similary, let $V_j(v)$ be the expected payoff of seller j conditional on his valuation being $v_j = v$, written as $V_j(v) = S_j(v) - vQ_j(v)$.

Lemma 1: In a direct revelation game U_i is convex and nondecreasing, $U_i' = P_i$ almost everywhere on U, and

$$R_i(u) - R_i(u^0) = \int_{u^0}^{u} t \, dP_i(t) \quad . \tag{3.1}$$

The sellers' expected payoffs have analogous properties.

Proof: In a direct revelation game buyer i's strategy has $\rho_i(\hat{u}) = \hat{u}$, and for this to be optimal requires that $U_i(\hat{u}) \geq uP_i(\hat{u}) - R_i(u)$ for every $u, \hat{u} \in U$. Collecting terms appropriately yields

$$U_i(\hat{u}) \geq U_i(u) + [\hat{u} - u] \cdot P_i(u) \quad ,$$

implying that U_i has a supporting hyperplane at u with slope $P_i(u) \geq 0$. Thus U_i is convex and nondecreasing with derivative $U_i'(u) = P_i(u)$ almost everywhere. This in turn implies that $R_i'(u) = uP_i'(u)$, which yields (3.1).

It is valuable to note that the relationship (1) between the agent's expected payment and his probability of trading has implications for more general games than direct revelation games. As noted in Section 2, a Nash equilibrium (ρ,σ) for a general trading rule τ induces a direct revelation game. Consequently, the properties in Lemma 1 are in general necessary conditions for a Nash equilibrium.

The trading rules encountered in practice have the property that each agent has a feasible strategy that ensures a nonnegative payoff. In a double auction, for instance, the identity function is a strategy that has this property; or, one could allow the possibility that an agent does not participate. In the corresponding induced direct revelation game this property implies that $U_i(u^0) \geq 0$ for each buyer i, and similarly $V_j(v^1) \geq 0$ for each seller j. Myerson and Satterthwaite [1981] obtain a remarkably simple characterization of the trading rules that satisfy this requirement. Denote an allocation's assignment of items by a subset $A \subseteq M \cup N$, indicating that those buyers $i \in A$ receive items from those sellers $j \in A$. Recall that a trading rule τ specifies a probability distribution over such assignments contingent on each combination (r,s) of the agents' actions.

<u>Lemma 2</u>: In a direct revelation game

$$E\{\sum_{i\in A}[u_i - [1 - F(u_i)]/f(u_i)] - \sum_{j\in A}[v_j + G(v_j)/g(v_j)]\}$$

$$= \sum_i U_i(u^0) + \sum_j V_j(v^1) \quad ,$$

where E here denotes expectation with respect to u, v, A.

<u>Proof</u>: Observe first that:

$$E\{\sum_{i\in A} u_i - \sum_{j\in A} v_j\} = \sum_i E\{U_i(u_i)\} + \sum_j E\{V_j(v_j)\}$$

Consequently, it sufficies to construct the buyers' terms:

$$\sum_i E\{U_i(u_i)\} = \sum_i E\{U_i(u^0) + \int_{u^0}^{u_i} P_i(t)dt\}$$

$$= \sum_i U_i(u^0) + \sum_i E\{[1 - F(u_i)]/f(u_i) \cdot P_i(u_i)\}$$

$$= \sum_i U_i(u^0) + E\{\sum_{i\in A}[1 - F(u_i)]/f(u_i)\} \quad .$$

Here, the first equality uses Lemma 1. The sellers' terms are constructed similarly.

The property that each agent obtains a nonnegative expected payoff is called <u>individual rationality</u>. In the sequel we require that a trading rule has this property. In view of Lemmas 1 and 2, a direct revelation game satisfies individual rationality only if the inequality

$$(3.2) \qquad E\{\sum_{i\in A}[u_i - [1 - F(u_i)]/f(u_i)] - \sum_{j\in A}[v_j + G(v_j)/g(v_j)]\} > 0$$

is satisfied. Since a Nash equilibrium for any trading rule induces a direct revelation game, this inequality is also necessary for individual rationality in the general case. In the sequel we consider only cases in which (3.2) is an equality.

The study of efficient trading rules is facilitated considerably by Lemma 2. Consider the problem of designing a trading rule for which the expected gains from trade are maximized. Of course in comparing two trading rules one evaluates the agents' expected payoffs using their Nash equilibrium strategies.

Lemma 3: A direct revelation game with trading rule τ is efficient (and maximizes the expected gains from trade) if there exists a number $\lambda > 0$ such that $\tau(r,s)\{A\} > 0$ only if

$$(3.3) \qquad A \in \arg\max_{A'} \sum_{i \in A'} [(1+\lambda)\cdot r_i - \lambda \cdot [1 - F(r_i)]/f(r_i)]$$

$$- \sum_{j \in A'} [(1+\lambda)\cdot s_j + \lambda \cdot G(s_j)/g(s_j)] \quad .$$

Proof: Such a trading rule maximizes the expected gains from trade,

$$E\{\sum_{i \in A} u_i - \sum_{j \in A} v_j\} \quad ,$$

subject to the constraint (3.2), for which λ is a Lagrange multiplier.

The interpretation of Lemma 3 in the context of more general revelation games provides the main tool used in the analysis of double auctions in the next section.

Theorem 1: A double auction for which the Nash equilibrium strategies are

$$\rho_i(u) = \phi(u - \alpha \cdot [1 - F(u)]/f(u))$$

and

$$\sigma_j(v) = \phi(v + \alpha \cdot G(v)/g(v)) \quad ,$$

for some α [0,1] and some increasing function ϕ, is an efficient trading rule (and maximizes the expected gains from trade).

Proof: Recall that a double auction chooses

$$A \in \arg\max_{A'} \sum_{i \in A'} r_i - \sum_{j \in A'} s_j \quad .$$

If the agents' strategies have the indicated form, and $\alpha = \lambda/[1 + \lambda]$, then this assignment is precisely the same as the one chosen in (3.3); that is, the selection of the assignment depends only on ordinal comparisons that are unaffected by the transformation ϕ.

4. An Example of a Double Auction

In this section we study an especially simple example of a double auction. The analysis of this example is guided by Theorem 1. Our aim is to verify that for this example a double auction is an efficient trading process. To do this we confirm that there exists a Nash equilibrium satisfying the sufficient condition specified in Theorem 1.

The special features of the example are that the numbers of buyers and sellers are the same $(m = n)$, and the agents' valuations are all uniformly distributed on the same interval. Thus we can take $U = V = [0,1]$ and then $F(u) = u$ and $G(v) = v$. In this case Theorem 1 can be interpreted as saying that if there is a Nash equilibrum of the form

$$\rho_i(u) = \psi(u - \delta) \quad \text{and} \quad \sigma_j(v) = \psi(v + \delta)$$

for some number $\delta \in [0,1/4]$ and some increasing function ψ, then the double auction is an efficient trading rule for this example. (Here, $\delta = .5\alpha/[1 + \alpha]$ and $\psi(x) = \phi([x + .5\alpha]/[1 + \alpha])$.) Initially we shall also adopt the hypothesis that ψ has the property that $\psi(x) + \psi(1 - x) = 1$ and then verify later that this property is satisfied. Further, as a technical matter it is well to realize that there is more than one function ψ that will provide the requisite Nash equilibrium. We will seek one that is differentiable and characterize it via a differential equation. However, typically this solution will have the unfortunate property that $\rho_i(u) > u$ if $u < 2\delta$, and analogously for the sellers. This is inconsequential for the Nash equilibrium, since no trade occurs in this situation, but in practice it is better to take the so-called <u>perfect</u> Nash equilibrium in which $\rho_i(u) = \min\{u, \psi(u - \delta)\}$ so that a buyer is not upset to find that a seller has used some strategy other than the one hypothesized in the equilibrium.

Our method is as follows. We first analyze the decision problem of a typical buyer i, and then indicate briefly the results of a

corresponding analysis of the decision problem of a typical seller. For the typical buyer we first suppose that each other buyer and the sellers are all using strategies of the form indicated in the hypothesized Nash equilibrium, and then determine which strategy is his optimal response. This best-response strategy will indeed be the same one specified in the Nash equilibrium if ψ satisfies a certain differential equation. As it turns out, this differential equation is the same as the one implied by the analysis of a typical seller's decision problem, and the solution ψ has the requisite properties. We then deal with a few loose ends, such as the determination of the right value of δ.

The following notation is helpful. Let $r(k)$ be the k-th largest bid among the buyers <u>except</u> buyer i, using the conventions that $r(0) = \infty$ and $r(m) = -\infty$. Defining

$$Y_k = \min\{r(k-1), s[k+1]\}$$

and

$$Z_k = \min\{r(k-1), s[k]\} \quad ,$$

let $K = \arg\max_k Z_k$ and set

$$Y = \max\{Y_{K-1}, Y_K\} \quad , \quad Z = Z_K \quad .$$

One can check that $Y > Z$ and that $Y = Z$ only if $s[K] = r(K-1)$ or $s[K+1]$.

We will use the property of a double auction that buyer i trades if and only if $r_i > Z$. That is, if i is to trade it must be for

every k that if $r_i < r(k-1)$ then $r_i > s[k]$. Furthermore, though it is somewhat tedious to explain, if $Y > r_i > Z$ then he trades at the price $p = [r_i + Z]/2$, and if $r_i > Y$ then he trades at the price $p = [Y + Z]/2$. For example, if $Z = s[K]$ then $Y = Y_K$ and $Z = Y_{K-1}$, and the configuration is necessarily

$$r(K) < s[K] < r_i < \min\{r(K-1), s[K+1]\} \quad ;$$

hence, there are K trades and i is the marginal buyer. Similarly, if $Z = r(K-1)$ then $Z = Y_K$ and $Y = Y_{K-1}$, and the configuration is

$$s[K-1] < r(K-1) < r_i < \min\{r(K-2), s[K]\} \quad ;$$

hence, there are $K - 1$ trades and again i is the marginal buyer. Now due to the hypothesized form of the Nash equilibrium strategies this amounts to saying that he trades at the first of these prices if $\psi(y) > r_i > \psi(z)$, and at the second if $r_i > \psi(y)$, where

$$\psi^{-1}(Z) \equiv z = \max_k \min\{u(k-1) - \delta, v[k] + \delta\}$$

and analogously for y, using corresponding definitions in terms of the rank ordered values of the other buyers' valuations$-\ \delta$ and the sellers' valuations$+\ \delta$. Thus, if we let $\hat{H}(y,z;\delta)$ be the joint distribution function of the pair (y,z) (on the support $\{y > z\}$) then buyer i's expected payoff when his valuation is u and he bids r is

$$\int_{z<\psi^{-1}(r)} u \, d\hat{H}(y,z;\delta) - \int_{y<\psi^{-1}(r)} [\psi(y) + \psi(z)]/2 \, d\hat{H}(y,z;\delta)$$

$$- \int_{z<\psi^{-1}(r)<y} [r + \psi(z)]/2 \, d\hat{H}(y,z;\delta) \quad .$$

To verify the Nash equilibrium we must show that $\rho_1(u)$ is the value of r that maximizes this expected payoff.

To simplify notation, let $H(x;\delta) = \Pr\{z < x < y\}$ and $h(x;\delta) = \Pr\{z = x < y\}$; that is

$$H(x;\delta) = \int_{z<x<y} d\hat{H}(y,z;\delta)$$

and

$$h(x;\delta) = \int_{x<y} d\frac{\partial\hat{H}}{\partial z}(y,x;\delta) \quad .$$

Note too that $\partial\psi^{-1}(r)/\partial r = 1/\psi'(x)$ if $r = \psi(x)$. Consequently, the first-order necessary condition for the optimal choice of r is

$$[u - r] \cdot h(\psi^{-1}(r);\delta) \cdot \frac{\partial\psi^{-1}(r)}{\partial r} = \frac{1}{2} \cdot H(\psi^{-1}(r);\delta) \quad .$$

At the required value $r = \psi(u - \delta)$ this is just the differential equation

$$(4.1) \qquad [u - \psi(u-\delta)] \cdot h(u-\delta;\delta) = \frac{1}{2} \cdot H(u-\delta;\delta) \cdot \psi'(u-\delta) \quad ,$$

which characterizes what the function ψ must be in order to sustain the hypothesized Nash equilibrium.

Turning to the decision problem of a typical seller, a similar analysis invokes an analogous set of definitions of Y, Z, <u>et cetera</u>, except that the roles of the buyers and sellers are reversed, as are the <u>max</u> and <u>min</u> operators. The symmetry of the formulation and the fact that the valuations are all uniformly distributed imply, however, that

the corresponding random variables "y" and "z" defined for a seller (denote them by $\hat{y}$ and $\hat{z}$) have the property that the pair $(1 - \hat{y}, 1 - \hat{z})$ has precisely the same probability distribution as does the pair (y,z) defined for a buyer. Consequently, the first-order necessary condition for the typical seller's optimal offer yields the following differential equation that ψ must satisfy in order to sustain the Nash equilibrium:

$$(4.2) \qquad [\psi(v + \delta) - v] \cdot h(1 - (v + \delta); \delta)$$
$$= \frac{1}{2} \cdot H(1 - (v + \delta); \delta) \cdot \psi'(v + \delta) \quad .$$

If $\psi(x) + \psi(1 - x) = 1$ as we supposed initially, then (4.1) and (4.2) are identical, since u and $1 - v$ have identical roles. With this proviso, therefore, we have confirmed that there exists a Nash equilibrium of the required form, and that ψ is given by a particular solution to the differential equation (4.1).

We can now resolve the loose ends in the argument. First, the particular solution that is to be chosen is determined by the condition that $\psi(\delta) = 2\delta$. That is, considering a buyer i, there is no chance of trading if $u_i < 2\delta$ so $U_i(2\delta) = U_i(u^0) = 0$, and to ensure that $\lim_{u \downarrow 2\delta} U_i(u) = 0$ requires that $\rho_i(2\delta) = 2\delta$. The symmetric condition for a seller, $\psi(1 - \delta) = 1 - 2\delta$, then determines the appropriate value of δ. If $n = 1$ then (as we shall see below) $\delta = 1/8$, and presumably δ is a declining function of n, so we will take it for granted here that $\delta \in [0,1/4)$ as required. The proviso that $\psi(x) + \psi(1 - x) = 1$

when $x \neq \delta$ or $1 - \delta$ is resolved purely on the grounds of symmetry between the buyers and the sellers, since throughout the roles of u and $1 - v$ are symmetric with appropriate reversals of the max and min operators. Lastly we mention that the first-order necessary condition for an optimal bid that was used in the construction can be shown to be a sufficient condition: one employs the properties of affiliated random variables developed by Milgrom and Weber [1982] to show that H and h have the required properties, and that ψ is increasing as required.

To illustrate the preceding analysis we apply it to the case $n = 1$ studied by Chatterjee and Samuelson [1979]. In this case $Y_0 = s_1$, $Y_1 = \infty$, and $Z_1 = s_1$, so $Y = \infty$ and $Z = s_1$. Similarly, $y = \infty$ and $z = v_1 + \delta$, so $H(x; \delta) = x - \delta$ and $h(x; \delta) = 1$ if $x < 1 - \delta$. Thus, with $x = u - \delta$ the differential equation (3.4) is

$$x + \delta - \psi = \frac{1}{2}(x - \delta)\psi' \quad ,$$

for which the solution is

$$\psi(x) = \frac{2[x^3/3 - \delta^2 x] + C}{[x - \delta]^2} \quad ,$$

for any particular choice of the constant C. Requiring that $\psi(\delta) = 2\delta$ determines the choice $C = 4\delta^3/3$ and yields $\psi(x) = [2x + 4\delta]/3$. Requiring further that $\psi(1 - \delta) = 1 - 2\delta$ fixes $\delta = 1/8$. Thus, the final form is $\psi(x) = [4x + 1]/6$ and the equilibrium strategies are $\rho_1(u) = [8u + 1]/12$ and $\sigma_j(v) = [8v + 3]/12$.

There is an alternative interpretation of the construction that emphasizes the role of Lemma 1. Let β and γ be the functions $\beta(u) = u - \delta$ and $\gamma(v) = v + \delta$. We have constructed the buyers' and sellers' strategies from the composition $(\rho, \sigma) = \psi \circ (\beta, \gamma)$, having derived ψ from an analysis of the requirements for a Nash equilibrium. Another route is to apply the trading rule τ directly to get the outcome given by the composition $\tau \circ (\beta, \gamma)$, with which there is associated an imputed price π. However, this imputed price will not satisfy the requirements of incentive compatibility imposed by (3.1). One seeks, therefore, a transformation μ yielding the actual price $p = \mu(\pi)$ that conforms to (3.1). Then ψ can be identified from the commutative property $\tau \circ \psi = \mu \circ \tau$. The figure below depicts the commutative diagram, using ι to represent the identity map:

$$\begin{tikzcd} {} \arrow[r, "(\beta,\gamma)"] \arrow[d, "\iota"'] & {} \arrow[r, "\tau"] \arrow[d, "\psi"] & {} \arrow[d, "\mu"] \\ {} \arrow[r, "(\rho,\sigma)"] & {} \arrow[r, "\tau"] & {} \end{tikzcd}$$

Figure 1: Commutative Diagram

In the case $n = 1$ considered above, $\mu = \psi$ since they are linear functions.

5. Many Buyers and Sellers

The connection between the Walrasian equilibrium and the Nash equilibrium of the double auction appears clearly in the example studied

in Section 4. The Walrasian equilibrium supposes, in effect, that each buyer or seller bids or offers his true valuation. Let π be the Walrasian clearing price, and let p be the clearing price in the Nash equilibrium. Then one would have $p \approx \psi(\pi)$ if the quantity traded were to be the same (and ψ is nearly linear), but in general the fact that $\delta > 0$ will entail fewer trades in the Nash equilibrium than is supposed in the Walrasian equilibrium.

The plausible conjecture is that the Walrasian and Nash equilibria are approximately the same if there are many buyers and sellers. We shall show, in fact, that for the example in Section 4 one gets $\delta \to 0$ and $\psi(x) \to x$ as $n \to \infty$.

The differential equation (4.1) admits the integrating factor

$$\Theta(x; \delta) = e^{2\int_0^x \theta(t; \delta)dt} ,$$

where $x = u - \delta$ and $\theta = h/H$. Consequently, the solution has the form

$$\psi(x) = \delta + x - \int_\delta^x \frac{\Theta(t; \delta)}{\Theta(x; \delta)}dt ,$$

and one determines δ from the condition that $\psi(1 - \delta) = 1 - 2\delta$:

$$\delta = \frac{1}{2}\int_\delta^{1-\delta} \frac{\Theta(t; \delta)}{\Theta(1 - \delta; \delta)}dt .$$

One sees, therefore, that if $n \to \infty$ implies $\Theta(t; \delta)/\Theta(x; \delta) \to 0$ for $t < x$, then $\delta \to 0$ and $\psi(x) \to x$, as required. It suffices, therefore, to show that $\theta(x; \delta) \to \infty$. In the present case

$$H(x;\delta)$$

$$= \sum_{k=1}^{n} \binom{n}{k}\binom{n-1}{k-1}[x-\delta]^k[1-(x-\delta)]^{n-k}[x+\delta]^{n-k}[1-(x+\delta)]^{k-1}$$

and

$$h(x;\delta) = \sum_{k=1}^{n} \binom{n}{k}\binom{n-1}{k-1}\left[\frac{k}{x-\delta} + \frac{n-k}{x+\delta}\right][x-\delta]^k \cdot [1-(x-\delta)]^{n-k}[x+\delta]^{n-k}[1-(x+\delta)]^{k-1} .$$

Thus, $\theta > n$, which is sufficient to prove the result. Of course p and π converge to 1/2 almost surely.

For this example, therefore, the intuition that perfect competition, in the sense of many buyers and sellers, leads to the Walrasian equilibrium is confirmed.

6. Remarks

It remains an open question whether a double auction is an efficient trading process in more general circumstances than the example studied in Sections 4 and 5.[1] It is at least clear that in contexts in which the buyers and sellers are asymmetrically situated, either because there are differing numbers of agents on the two sides of the market or because their valuations are distributed differently, the method of analysis will need to be altered. Here we have relied on Lemma 3 which treats the agents symmetrically by taking the gains from trade as the measure of welfare. In the general case the welfare function imputed from a double auction is likely to assign differing weights to the

various agents. Moreover, adopting ex ante welfare maximization as the efficiency criterion may be unduly strong. If one adopts the weaker criterion of interim efficiency developed in the work of Holmström and Myerson [1981] then the appropriate criterion is to maximize

$$E\{\sum_i w_i(u_i)U_i(u_i) + \sum_j w_j(v_j)V_j(v_j)\} \quad ,$$

for some system $w_i(\cdot)$ and $w_j(\cdot)$ of contingent weights. Using (3.1) this criterion can be put in a tractable form to produce analogues of Lemma 3 and Theorem 1, but we do not pursue this route any further here.

The sort of exercise pursued in Section 5 lies at the heart of the problem of unravelling the foundations of the Walrasian model of markets. Surely the justification for the Walrasian model lies in the hidden conjecture that when there are many agents on both sides of the market each agent's optimal strategy is very nearly to bid or offer his privately known valuation. Only if this conjecture is true can one suppose that aggregate demand and supply curves constructed from agents' bids and offers reflect truly their preferences.

Footnotes

1/ An easy generalization assumes that $m = n$ and $F(x) + G(1 - x) = 1$, but I have not succeeded in solving any cases that are genuinely asymmetric.

References

Chatterjee, Kalyan and William Samuelson [1979], "The Simple Economics of Bargaining," Technical Report, Pennsylvania State University.

Easley, David and John Ledyard [1981], "A Theory of Price Formation and Exchange in Oral Auctions," Technical Report, J. L. Kellogg Graduate School of Management, Northwestern University.

Holmström, Bengt and Roger B. Myerson [1981], "Efficient and Durable Decision Rules with Incomplete Information," Technical Report, J. L. Kellogg Graduate School of Management, Northwestern University.

Milgrom, Paul R. and Robert J. Weber [1982], "A Theory of Auctions and Competitive Bidding," Econometrica, Vol. 50, 1089-1122.

Myerson, Roger B. and Mark A. Satterthwaite [1981], "Efficient Mechanisms for Bilateral Trading," Technical Report, J. L. Kellogg Graduate School of Management, Northwestern University.

Plott, Charles R. and Vernon L. Smith [1978], "An Experimental Examination of Two Exchange Institutions," Review of Economic Studies, Vol. 45, 133-153.

Smith, Vernon L. [1981], "Microeconomic Systems as an Experimental Science," Technical Report, University of Arizona.

Smith, Vernon L., Arlington W. Williams, W. Kenneth Bratton, and Michael G. Vannoni [1982], "Competitive Market Institutions: Double Auctions vs. Sealed Bid-Offer Auctions," The American Economic Review, Vol. 72, 58-77.

REPORTS IN THIS SERIES

160. "The Structure and Stability of Competitive Dynamical Systems," by David Cass and Karl Shell.
161. "Monopolistic Competition and the Capital Market," by J. E. Stiglitz.
162. "The Corporation Tax," by J. E. Stiglitz.
163. "Measuring Returns to Scale in the Aggregate and the Scale Effect of Public Goods," by David A. Starrett.
164. "Monopoly, Quality, and Regulation," by Michael Spence.
165. "A Note on the Budget Constraint in a Model of Borrowing," by Duncan K. Foley and Martin F. Hellwig.
166. "Incentives, Risk, and Information: Notes Towards a Theory of Hierarchy," by Joseph E. Stiglitz.
167. "Asymptotic Expansions of the Distributions of Estimates in Simultaneous Equations for Alternative Parameter Sequences," by T. W. Anderson.
168. "Estimation of Linear Functional Relationships: Approximate Distributions and Connections with Simultaneous Equations in Econometrics," by T. W. Anderson.
169. "Monopoly and the Rate of Extraction of Exhaustible Resources," by Joseph E. Stiglitiz.
170. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," by Michael Rothschild and Joseph Stiglitz.
171. "Strong Consistency of Least Squares Estimates in Normal Linear Regression," by T. W. Anderson and John B. Taylor.
172. "Incentive Schemes under Differential Information Structures: An Application to Trade Policy," by Partha Dasgupta and Joseph Stiglitz.
173. "The Incidence and Efficiency Effects of Taxes on Income from Capital," by John B. Shoven.
174. "Distribution of a Maximum Likelihood Estimate of a Slope Coefficient: The LIML Estimate for Known Covariance Matrix," by T. W. Anderson and Takamitsu Sawa.
175. "A Comment on the Test of Overidentifying Restrictions," by Joseph B. Kadane and T. W. Anderson.
176. "An Asymptotic Expansion of the Distribution of the Maximum Likelihood Estimate of the Slope Coefficient in a Linear Functional Relationship," by T. W. Anderson.
177. "Some Experimental Results on the Statistical Properties of Least Squares Estimates in Control Problems," by T. W. Anderson and John B. Taylor.
178. "A Note on "Fulfilled Expectations" Equilibria," by David M. Kreps.
179. "Uncertainty and the Rate of Extraction under Alternative Institutional Arrangements," by Partha Dasgupta and Joseph E. Stiglitz.
180. "Budget Displacement Effects of Inflationary Finance," by Jerry Green and E. Sheshinski.
181. "Towards a Marxist Theory of Money," by Duncan K. Foley.
182. "The Existence of Futures Markets, Noisy Rational Expectations and Informational Externalities," by Sanford Grossman.
183. "On the Efficiency of Competitive Stock Markets where Traders have Diverse Information," by Sanford Grossman.
184. "A Bidding Model of Perfect Competition," by Robert Wilson.
185. "A Bayesian Approach to the Production of Information and Learning by Doing," by Sanford J. Grossman, Richard E. Kihlstrom and Leonard J. Mirman.
186. "Disequilibrium Allocations and Recontracting," by Jean-Michel Grandmont, Guy Laroque and Yves Younes.
187. "Agreeing to Disagree," by Robert J. Aumann.
188. "The Maximum Likelihood and the Nonlinear Three Stage Least Squares Estimator in the General Nonlinear Simultaneous Equation Model," by Takeshi Amemiya.
189. "The Modified Second Round Estimator in the General Qualitative Response Model," by Takeshi Amemiya.
190. "Some Theorems in the Linear Probability Model," by Takeshi Amemiya.
191. "The Bilinear Complementarity Problem and Competitive Equilibria of Linear Economic Models," by Robert Wilson.
192. "Noncooperative Equilibrium Concepts for Oligopoly Theory," by L. A. Gerard-Varet.
193. "Inflation and Costs of Price Adjustment," by Eytan Sheshinski and Yoram Weiss.
194. "Power and Taxes in a Multi-Commodity Economy," by R. J. Aumann and M. Kurz.
195. "Distortion of Preferences, Income Distribution and the Case for a Linear Income Tax," by Mordecai Kurz.
196. "Search Strategies for Nonrenewable Resource Deposits," by Richard J. Gilbert.
197. "Demand for Fixed Factors, Inflation and Adjustment Costs," by Eytan Sheshinski and Yoram Weiss.
198. "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersions," by Steve Salop and Joseph Stiglitz.
199. "The Design of Tax Structure: Direct Versus Indirect Taxation by A. B. Atkinson and J. E. Stiglitz.
200. "Market Allocations of Location Choice in a Model with Free Mobility," by David Starrett.
201. "Efficiency in the Optimum Supply of Public Goods," by Lawrence J. Lau, Eytan Sheshinski and Joseph E. Stiglitz.
202. "Risk Sharing, Sharecropping and Uncertain Labor Markets," by David M. G. Newberry.
203. "On Non-Walrasian Equilibria," by Frank Hahn.
204. "A Note on Elasticity of Substitution Functions," by Lawrence J. Lau.
205. "Quantity Constraints as Substitutes for Inoperative Markets: The Case of the Credit Markets," by Mordecai Kurz.
206. "Incremental Consumer's Surplus and Hedonic Price Adjustment," by Robert D. Willig.
207. "Optimal Depletion of an Uncertain Stock," by Richard Gilbert.
208. "Some Minimum Chi-Square Estimators and Comparisons of Normal and LogisticaModels in Qualitative Response Analysis," by Kumio Morimune.
209. "A Characterization of the Optimality of Equilibrium in Incomplete Markets," by Sanford J. Grossman.
210. "Inflation and Taxes in a Growing Economy with Debt and Equity Finance," by M. Feldstein, J. Green and E. Sheshinski.
211. "The Specification and Estimation of a Multivariate Logit Model," by Takeshi Amemiya.
212. Prices and Queues as Screening Devices in Competitive Markets," by Joseph E. Stiglitz.
213. "Conditions for Strong Consistency of Least Squares Estimates in Linear Models," by T. W. Anderson and John B. Taylor.
214. "Utilitarianism and Horizontal Equity: The Case for Random Taxation," by Joseph E. Stiglitz.
215. "Simple Formulae for Optimal Income Taxation and the Measurement of Inequality," by Joseph E. Stiglitz.
216. "Temporal Resolution of Uncertainty and Dynamic Choice Behavior," by David M. Kreps and Evan L. Porteus.
217. "The Estimation of Nonlinear Labor Supply Functions with Taxes from a Truncated Sample," by Michael Hurd.
218. "The Welfare Implications of the Unemployment Rate," by Michael Hurd.
219. "Keynesian Economics and General Equilibrium Theory: Reflections on Some Current Debates," by Frank Hahn.
220. "The Core of an Exchange Economy with Differential Information," by Robert Wilson.
221. "A Competitive Model of Exchange," by Robert Wilson.
222. "Intermediate Preferences and the Majority Rule," by Jean-Michel Grandmont.
223. "The Fixed Price Equilibria: Some Results in Local Comparative Statics," by Guy Laroque.
224. "On Stockholder Unanimity in Making Production and Financial Decisions," by Sanford J. Grossman and Joseph E. Stiglitz.
225. "Selection of Regressors," by Takeshi Amemiya.
226. "A Note on A Random Coefficients Model," by Takeshi Amemiya.
227. "A Note on a Heteroscedastic Model," by Takeshi Amemiya.
228. "Welfare Measurement for Local Public Finance," by David Starrett.
229. "Unemployment Equilibrium with Rational Expectations," by W. P. Heller and R. M. Starr.
230. "A Theory of Competitive Equilibrium in Stock Market Economies," by Sanford J. Grossman and Oliver D. Hart.
231. "An Application of Stein's Methods to the Problem of Single Period Control of Regression Models," by Asad Zaman.
232. "Second Best Welfare Economics in the Mixed Economy," by David Starrett.
233. "The Logic of the Fix-Price Method," by Jean-Michel Grandmont.
234. "Tables of the Distribution of the Maximum Likelihood Estimate of the Slope Coefficient and Approximations," by T. W. Anderson and Takamitsu Sawa.
235. "Further Results on the Informational Efficiency of Competitive Stock Markets," by Sanford Grossman.
236. "The Estimation of a Simultaneous-Equation Tobit Model," by Takeshi Amemiya.
237. "The Estimation of a Simultaneous-Equation Generalized Probit Model," by Takeshi Amemiya.
238. "The Consistency of the Maximum Likelihood Estimator in a Disequilibrium Model," by T. Amemiya and G. Sen.
239. "Numerical Evaluation of the Exact and Approximate Distribution Functions of the Two-Stage Least Squares Estimate," by T. W. Anderson and Takamitsu Sawa.
240. "Risk Measurement of Public Projects," by Robert Wilson.
241. "On the Capitalization Hypothesis in Closed Communities," by David Starrett
242. "A Note on the Uniqueness of the Representation of Commodity-Augmenting Technical Change," by Lawrence J. Lau
243. "The Property Rights Doctrine and Demand Revelation under Incomplete Information," by Kenneth J. Arrow.
244. "Optimal Capital Gains Taxation Under Limited Information," by Jerry R. Green and Eytan Sheshinski
245. "Straightforward Individual Incentive Compatibility in Large Economies," by Peter J. Hammond
246. "On the Rate of Convergence of the Core," by Robert J. Aumann.
247. "Unsatisfactory Equilibria," by Frank Hahn.
248. "Existence Conditions for Aggregate Demand Functions: The Case of a Single Index," by Lawrence J. Lau
249. "Existence Conditions for Aggregate Demand Functions: The Case of Multiple Indexes," by Lawrence J. Lau
250. "A Note on Exact Index Numbers," by Lawrence J. Lau.
251. "Linear Regression Using Both Temporally Aggregated and Temporally Disaggregated Data," by Cheng Hsiao
252. "The Existence of Economic Equilibria: Continuity and Mixed Strategies," by Partha Dasgupta and Eric Maskin
253. "A Complete Class Theorem for the Control Problem and Further Results on Admissibility and Inadmissibility," by Asad Zaman
254. "Measure-Based Values of Market Games," by Sergiu Hart.
255. "Altruism as an Outcome of Social Interaction," by Mordecai Kurz.
256. "A Representation Theorem for 'Preference for Flexibility'," by David M. Kreps
257. "The Existence of Efficient and Incentive Compatible Equilibria with Public Goods," by Theodore Groves and John O. Ledyard.
258. "Efficient Collective Choice with Compensation," by Theodore Groves
259. "On the Impossibility of Informationally Efficient Markets," by Sanford J. Grossman and Joseph E. Stiglitz

REPORTS IN THIS SERIES

260. "Values for Games Without Sidepayments Some Difficulties With Current Concepts," by Alvin E. Roth
261. "Martingales and the Valuation of Redundant Assets," by J. Michael Harrison and David M. Kreps
262. "Autoregressive Modelling and Money Income Causality Detection," by Cheng Hsiao
263. "Measurement Error in a Dynamic Simultaneous Equations Model without Stationary Disturbances," by Cheng Hsiao.
264. "The Measurement of Deadweight Loss Revisited," by W. E. Diewert.
265. "The Elasticity of Derived Net Supply and a Generalized Le Chatelier Principle," by W. E. Diewert.
266. "Income Distribution and Distortion of Preferences the ℓ Commodity Case," by Mordecai Kurz.
267. "The n^{-2} Order Mean Squared Errors of the Maximum Likelihood and the Minimum Logit Chi-Square Estimator," by Takeshi Amemiya.
268. "Temporal Von Neumann-Morgenstern and Induced Preferences," by David M. Kreps and Evan L. Porteus.
269. "Take-Over Bids and the Theory of the Corporation," by Stanford Grossman and Oliver D. Hart.
270. "The Numerical Values of Some Key Parameters in Econometric Models," by T. W. Anderson, Kimio Morimune and Takamitsu Sawa.
271. "Two Representations of Information Structures and their Comparisons," by Jerry Green and Nancy Stokey.
272. "Asymptotic Expansions of the Distributions of Estimators in a Linear Functional Relationship when the Sample Size is Large," by Naoto Kunitomo.
273. "Public Goods and Power," by R. J. Aumann, M. Kurz and A. Neyman.
274. "An Axiomatic Approach to the Efficiency of Non-Cooperative Equilibrium in Economies with a Continuum of Traders," by A. Mas-Colell.
275. "Tables of the Exact Distribution Function of the Limited Information Maximum Likelihood Estimator when the Covariance Matrix in Known," by T. W. Anderson and Takamitsu Sawa.
276. "Autoregressive Modeling of Canadian Money and Income Data," by Cheng Hsiao.
277. "We Can't Disagree Forever," by John D. Geanakoplos and Heraklis Polemarchakis.
278. "Constrained Excess Demand Functions," by Herklis M. Polemarchakis.
279. "On the Bayesian Selection of Nash Equilibrium," by Akira Tomioka.
280. "Disequilibrium Econometrics in Simultaneous Equations Systems," by C. Gourierous, J. J. Laffont and A. Monfort.
281. "Duality Approaches to Microeconomic Theory," by W. E. Diewert.
282. "A Time Series Analysis of the Impact of Canadian Wage and Price Controls," by Cheng Hsiao and Oluwalayo Fakiyesi.
283. "A Strategic Theory of Inflation," by Mordecai Kurz.
284. "A Characterization of Vector Measure Games in pNA," by Yair Tauman.
285. "On the Method of Taxation and the Provision of Local Public Goods," by David A. Starrett.
286. "An Optimization Problem Arising in Economics Approximate Solutions, Linearity, and a Law of Large Numbers," by Sergiu Hart.
287. "Asymptotic Expansions of the Distributions of the Estimates of Coefficients in a Simultaneous Equation System," by Yasunori Fujikoshi, Kimio Morimune, Naoto Kunitomo and Masanobu Taniguchi
288. "Optimal & Voluntary Income Distribution," by K. J. Arrow.
289. "Asymptotic Values of Mixed Games," by Abraham Neyman.
290. "Time Series Modelling and Causal Ordering of Canadian Money, Income and Interest Rate," by Cheng Hsiao.
291. "An Analysis of Power in Exchange Economies," by Martin J. Osborne.
292. "Estimation of the Reciprocal of a Normal Mean," by Asad Zaman.
293. "Improving the Maximum Likelihood Estimate in Linear Functional Relationships for Alternative Parameter Sequences," by Kimio Morimune and Naoto Kunitomo.
294. "Calculation of Bivariate Normal Integrals by the Use of Incomplete Negative-Order Moments," by Kei Takeuchi and Akimichi Takemura.
295. "On Partitioning of a Sample with Binary Type Questions in Lieu of Collecting Observations," by Kenneth J. Arrow, Leon Pesotchinsky and Milton Sobel.
296.
297. "The Two Stage Least Absolute Deviations Estimators," by Takeshi Amemiya.
298. "Three Essays on Capital Markets," by David M. Kreps
299. "Infinite Horizon Programs," by Michael J. P. Magill
300. "Electoral Outcomes and Social Log-Likelihood Maxima," by Peter Coughlin and Shmuel Nitzan.
301. "Notes on Social Choice and Voting," by Peter Coughlin.
302. "Overlapping Expectations and Hart's Conditions for Equilibrium in a Securities Model," by Peter J. Hammond.
303. "Directional and Local Electorate Competitions with Probabilistic Voting," by Peter Coughlin and Shmuel Nitzan.
304. "Asymptotic Expansions of the Distributions of the Test Statistics for Overidentifying Restrictions in a System of Simultaneous Equations," by Kunitomo, Morimune, and Tsukuda
305. "Incomplete Markets and the Observability of Risk Preference Properties," by H. H. Polemarchakis and L. Selden.
306. "Multiperiod Securities and the Efficient Allocation of Risk A Comment on the Black-Scholes Option Pricing Model," by David M. Kreps
307. "Asymptotic Expansions of the Distributions of k-Class Estimators when the Disturbances are Small," by Naoto Kunitomo, Kimio Morimune, and Yoshihiko Tsukuda.
308. "Arbitrage and Equilibrium in Economies with Infinitely Many Commodities," by David M. Kreps.
309. "Unemployment Equilibrium in an Economy with Linked Prices," by Mordecai Kurz
310. "Pareto Optimal Nash Equilibria are Competitive in a Repeated Economy," by Mordecai Kurz and Sergiu Hart.
311. "Identification," by Cheng Hsiao
312. "An Introduction to Two-Person Zero Sum Repeated Games with Incomplete Information," by Sylvain Sorin.
313. "Estimation of Dynamic Models with Error Components," by T. W. Anderson and Cheng Hsiao.
314. "On Robust Estimation in Certainty Equivalence Control," by Anders H. Westlund and Hans Stenlund.
315. "On Industry Equilibrium Under Uncertainty," by J. Dreze and E. Sheshinski.
316. "Cost Benefit Analysis and Project Evaluation from the Viewpoint of Productive Efficiency," by W. E. Diewert.
317. "On the Chain-Store Paradox and Predation: Reputation for Toughness," by D. M. Kreps and Robert Wilson.
318. "On the Number of Commodities Required to Represent a Market Games," Sergiu Hart.
319. "Evaluation of the Distribution Function of the Limited Information Maximum Likelihood Estimator," by T. W. Anderson, Naoto Kunitomo, and Takamitsu Sawa.
320. "A Comparison of the Logit Model and Normal Discriminant Analysis when the Independent Variables are Binary," by Takeshi Amemiya and James L. Powell.
321. "Efficiency of Resource Allocation by Uninformed Demand," by Theodore Groves and Sergiu Hart.
322. "A Comparison of the Box-Cox Maximum Likelihood Estimator and the Nonlinear Two Stage Least Squares Estimator," by Takeshi Amemiya and James L. Powell.
323. "Comparison of the Densities of the TSLS and LIMLK Estimators for Simultaneous Equations," by T. W. Anderson, Naoto Kunitomo, and Takamitsu Sawa.
324. "Admissibility of the Bayes Procedure Corresponding to the Uniform Prior Distribution for the Control Problem in Four Dimensions but Not in Five," by Charles Stein and Asad Zaman.
325. "Some Recent Developments on the Distributions of Single-Equation Estimators," by T. W. Anderson.
326. On Inflation," by Frank Hahn.
327. Two Papers on Majority Rule: "Continuity Properties of Majority Rule with Intermediate Preferences," by Peter Coughlin and Kuan-Pin Lin; and, "Electroal Outcomes with Probabilistic Voting and Nash Social Welfare Maxima," by Peter Coughlin and Shmuel Nitzan.
328. "On the Endogenous Formation of Coalitions," by Sergiu Hart and Mordecai Kurz.
329. "Controliability, Pecuniary Externalities and Optimal Taxation," by David Starrett.
330. "Nonlinear Regression Models," by Takeshi Amemiya.
331. "Paradoxical Results from Inada's Conditions for Majority Rule," by Herve Raynaud.
332. "On Welfare Economics with Incomplete Information and the Social Value of Public Information," by Peter J. Hammond.
333. "Equilibrium Policy Proposals with Abstentions," by Peter J. Coughlin.
334. "Infinite Excessive and Invariant Measures," by Michael I. Taksar.
335. "The Life-Cycle Hypothesis and the Effects of Social Security and Private Pensions on Family Savings," by Mordecai Kurz.
336. "Optimal Retirement Age," by Mordecai Kurz.
337. "Bayesian Incentive Compatible Beliefs," by Claude d'Aspremont and Louis-Andre Gerard-Varet.
338. "Qualitative Response Models: A Survey," by Takeshi Amemiya.
339. "The Social Costs of Monopoly and Regulation: A Game Theoretic Analysis," by William P. Rogerson.
340. "Sequential Equilibria," by David M. Kreps and Robert Wilson.
341. "Enhancing of Semigroups," by Michael I. Taksar.
342. "Formulation and Estimation of Dynamic Models Using Panel Data," by T. W. Anderson and Cheng Hsiao.
343. "Ex-Post Optimality as a Dynamically Consistent Objective for Collective Choice Under Uncertainty," by Peter Hammond.
344. "Three Lectures in Monetary Theory," by Frank H. Hahn.
345. "Socially Optimal Investment Rules in the Presence of Incomplete Markets and Other Second Best Distortions," by Frank Milne and David A. Starrett.
346. "Approximate Purification of Mixed Strategies," by Robert Aumann, Yitzhak Katznelson, Roy Radner, Robert W. Rosenthal, and Benjamin Weiss.
347. "Conditions for Transitivity of Majority Rule with Algorithmic Interpretations," by Herve Raynaud.
348. "How Restrictive Actually are the Value Restriction Conditions," by Herve Raynaud.
349. "Cournot Duopoly in the Style of Fulfilled Expectations Equilibrium," by William Novshek and Hugh Sonnenschein.
350. "Law of Large Numbers for Random Sets and Allocation Processes," by Zvi Artstein and Sergiu Hart.
351. "Risk Perception in Psychology and Economics," by Kenneth J. Arrow.
352. "Shrunken Predictors for Autoregressive Models," by Taku Yamamoto.
353. "Predation, Reputation, and Entry Deterrence," by Paul Milgrom and John Roberts.
354. "Social and Private Production Objectives in the Sequence Economy," by David Starrett.
355. "Recursive Rational Choice," by Alain Lewis.
356. "Least Absolute Deviations Estimation for Censored and Truncated Regression Models," by James Powell.
357. "Relatively Recursive Rational Choice," by Alain Lewis.
358. "A Theory of Auctions and Competitive Bidding," by Paul Milgrom and Robert Weber.
359. "Facing an Uncertain Future," by W. M. Gorman.
360. "The Individual Freedom Allowed by the Value Restriction Condition," by Herve Raynaud.
361. "Incomplete Resource Allocation Mechanisms," by P. J. Hammond.
362. "The Comparative Dynamics of Efficient Programs of Capital Accumulation and Resource Depletion," by W. E. Diewert and T. R. Lewis.
363. "Optimum Pricing Policy Under Stochastic Inflation," by E. Sheshinski and Y. Weiss.
364. "Capital Accumulation and the Characteristics of Private Intergenerational Transfers," by M. Kurz.
365. "Asymptotic Efficiency and Higher Order Efficiency of the Limited Information Maximum Likelihood Estimator in Large Econometric Models," by N. Kunitomo.
366. "Semi-Values of Political Economic Games," by A. Neyman.
367. "Non-Zero-Sum Two-Person Repeated Games with Incomplete Information," by S. Hart.
368. "Partially Generalized Least Squares and Two-Stage Least Squares and Two-Stage Least Squares Estimators," by T. Amemiya.
369. "Lipschitz Properties of Solution in Mathematical Programming," by B. Cornet and G. Laroque.
370. "Multiple Hypothesis Testing," by N. E. Savin.
371. "Team Theory and Decentralized Resource Allocation: An Example," by K. J. Arrow.
372. "Testing for Unit Roots: 2," by G. B. A. and Evans and N. E. Savin.
373. "Asymptotic Bias of the Least Squares Estimator for Multivariate Autoregressive Models," by T. Yamamoto and N. Kunitomo.
374. "The Asymptotic Normality of Two-Stage Least Absolute Deviations Estimators," by J. Powell.
375. "Rational Cooperation in the Finitely-Repeated Prisoner's Dilemma," by D. Kreps, P. Milgrom, J. Roberts, and R. Wilson.

Reports in this Series

376. "Necessary and Sufficient Conditions for Single-Peakedness Along a Linearly Ordered Set of Policy Alternatives" by P.J. Coughlin and M.J. Hinich.

377. "The Role of Reputation in a Regeated Agency Problem Involving Information Transmission" by W. P. Rogerson.

378. "Unemployment Equilibrium with Stochastic Rationing of Supplies" by Ho-mou Wu.

379. "Optimal Price and Income Regulation Under Uncertainty in the Model with One Producer" by M. I. Taksar.

380. "On the NTU Value" by Robert J. Aumann.

381. "Best Invariant Estimation of a Direction Parameter with Application to Linear Functional Relationships and Factor Analysis" by T. W. Anderson, C. Stein and A. Zaman.

382. "Informational Equilibrium" by Robert Kast.

383. "Cooperative Oligopoly Equilibrium" by Mordecai Kurz.

384. "Reputation and Product Quality" by William P. Rogerson.

385. "Auditing: Perspectives from Multiperson Decision Theory" By Robert Wilson.

386. "Capacity Pricing" by Oren, Smith and Wilson.

387. "Consequentialism and Rationality in Dynamic Choice Under Uncertainty" by P.J. Hammond.

388. "The Structure of Wage Contracts in Repeated Agency Models" by W. P. Rogerson.

389. "1982 Abraham Wald Memorial Lectures, Estimating Linear Statistical Relationships by T.W. Anderson.

390. "Aggregates, Activities and Overheads" by W.M. Gorman.

391. "Double Auctions" by Robert Wilson.

END

FILMED

6-83

DTIC